Local heights computations for quadratic Chabauty

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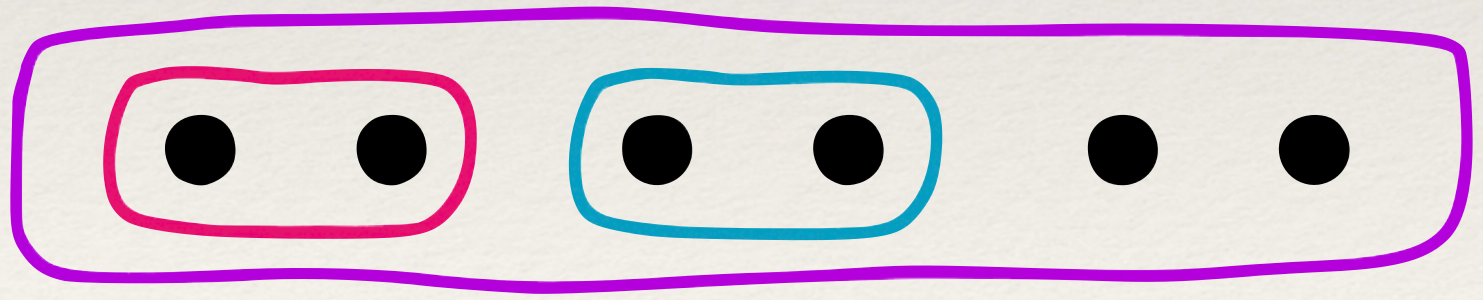
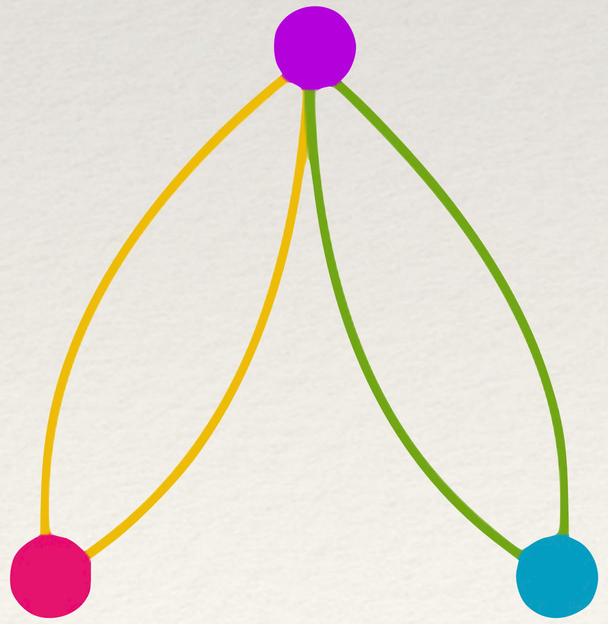
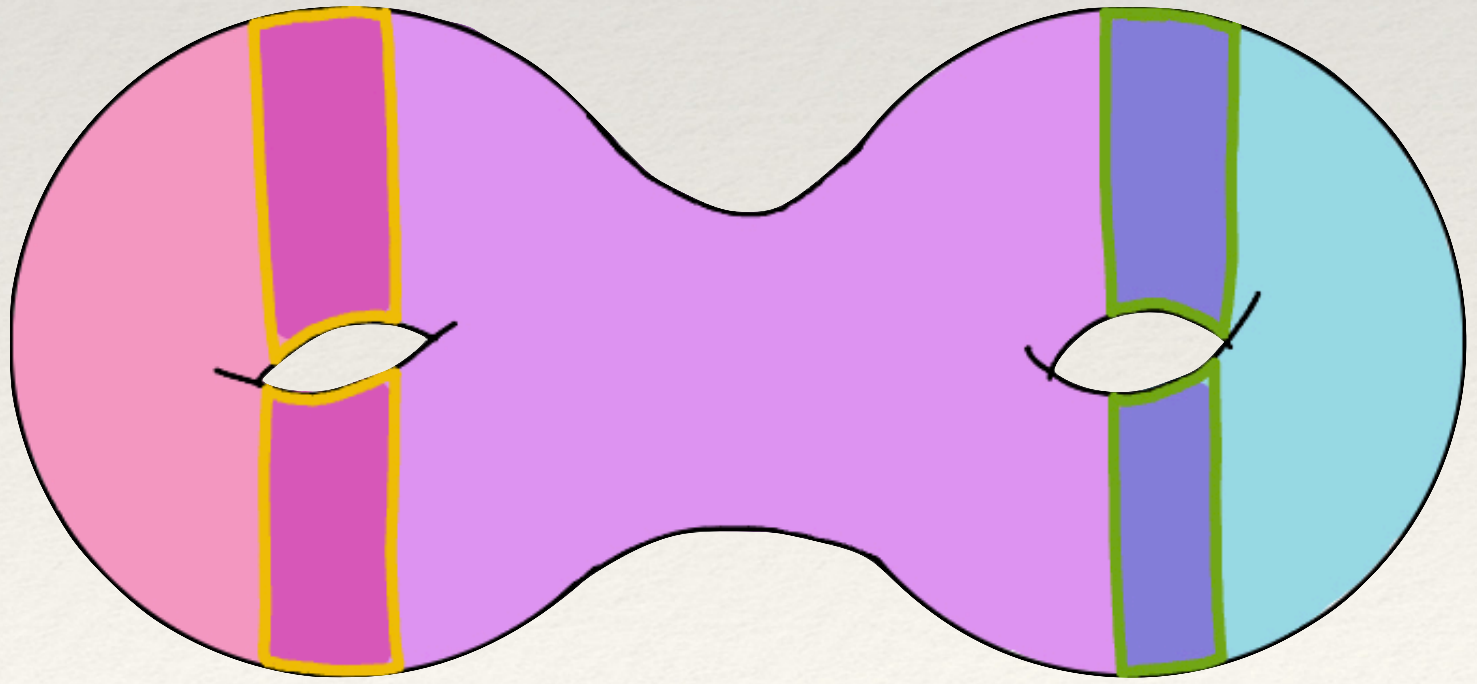
Joint work with Alexander Betts, Sachi Hashimoto, and Pim Spelier.







Local heights computations: why?

- Set-up: Let C be a nice curve of genus $g \geq 2$. Then $\#C(\mathbb{Q}) < \infty$.
- **Goal:** To describe explicitly $C(\mathbb{Q})$.
- Method: **quadratic Chabauty** (explicitly presented by Balakrishnan & Dogra, '18 '21). This is a p-adic method that has been successfully used to compute $C(\mathbb{Q})$ in many new cases.
- Key input: Let p be a prime and $Z \subset C \times C$ be a trace 0 correspondence. There is an associated p**-adic** (Coleman Gross) **height function** $h_Z : C(\mathbb{Q}) \to \mathbb{Q}_p$ which can be decomposed as

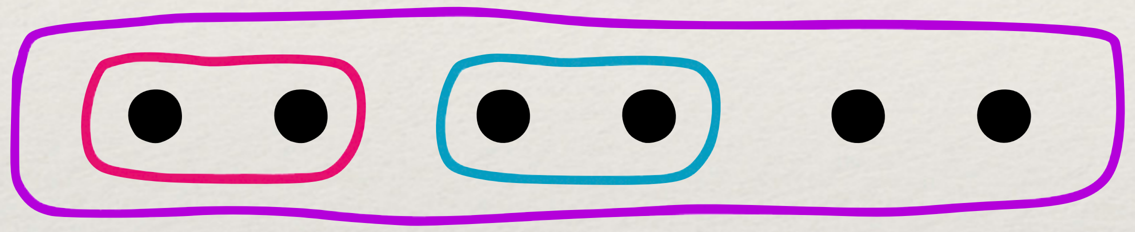
$$h_Z(Q) = \sum_{\ell} h_{Z,\ell}(Q),$$

  where $h_{Z,\ell} : C(\mathbb{Q}_\ell) \to \mathbb{Q}_p$.
- **One challenge:** Computing local heights.

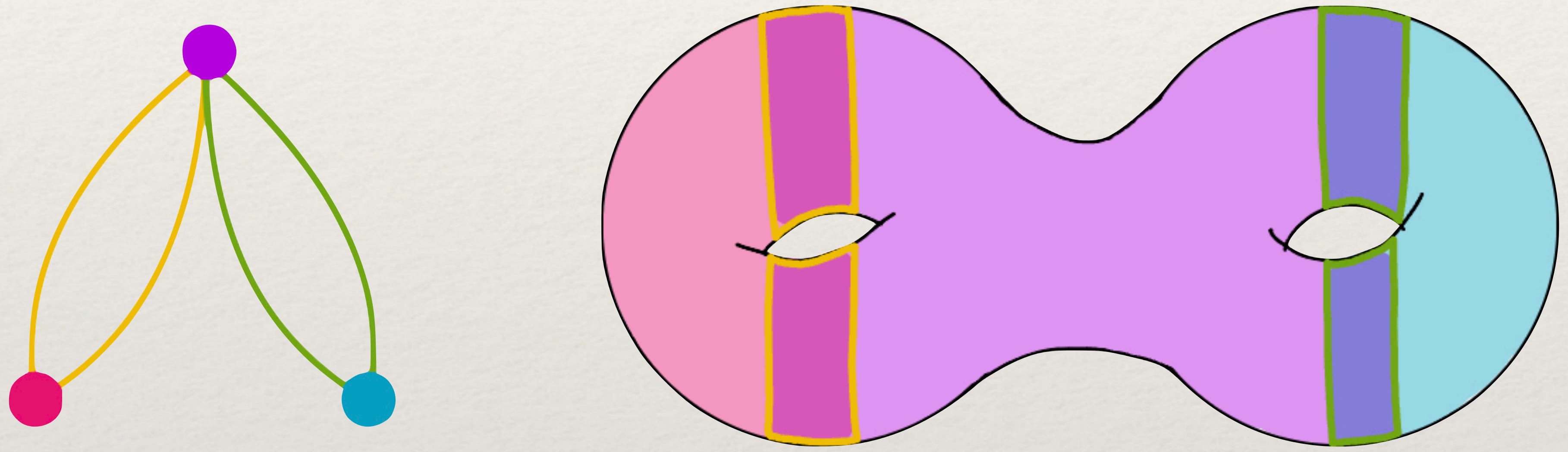
Local heights computations on hyperelliptic curves: how?

$$y^2 = x^6 + 2x^4 + 6x^3 + 5x^2 - 6x + 1$$

We pick a correspondence $Z \subset C \times C$ with action on $H^0(X, \Omega^1_X)$ given by $\begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$.



Cluster picture

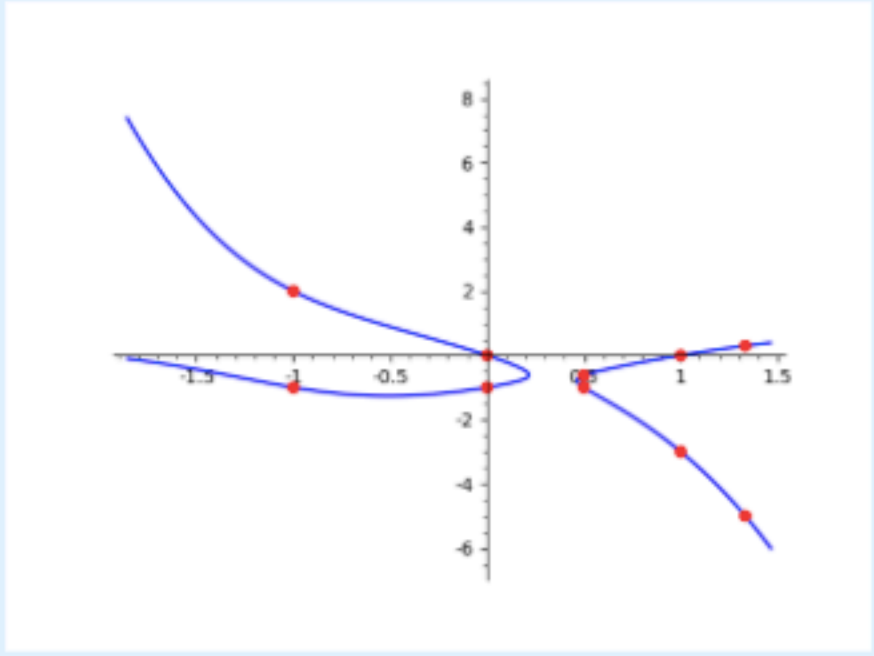


Berkovich space decomposition

$$h_{Z,3}(x,y) = \begin{cases} -\frac{1}{4}\log^p(3) & \text{if } x \equiv -1 \bmod 3, \\ +\frac{1}{4}\log^p(3) & \text{if } x \equiv +1 \bmod 3, \\ 0 & \text{otherwise.} \end{cases}$$

Properties

Label	8649.a.77841.1



Conductor	8649
Discriminant	77841
Mordell-Weil group	$\mathbb{Z} \oplus \mathbb{Z}$
Sato-Tate group	$\mathrm{SU}(2) \times \mathrm{SU}(2)$
$\mathrm{End}(J_{\overline{\mathbb{Q}}}) \otimes \mathbb{R}$	$\mathbb{R} \times \mathbb{R}$
$\mathrm{End}(J_{\overline{\mathbb{Q}}}) \otimes \mathbb{Q}$	RM
$\mathrm{End}(J) \otimes \mathbb{Q}$	RM

Prime	Cluster picture
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