Fermat Quotients, Contruction and Applications

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Fermat curves
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Quotients of Fermat curves
$F_k : v^p = u(1-u)^k$

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- A basis for the holomorphic one-forms of F_k is

$$\{\omega_{m,\langle mk \rangle} \,|\, 1 \leq m \leq p-1,\, 1 \leq m + \langle mk \rangle \leq p-1\}.$$

Set up:

- X is a complete non-singular genus $g > 1$ curve over K.
- $K = \overline{K}$ and $\mathrm{char}(K) = 0$.
- σ is an automorphism of X of order $N \geq 2g + 1$.
- $H_\lambda : y^2 = (x^{g+1} - 1)(x^{g+1} - \lambda)$, $\lambda \in K \setminus \{0, 1\}$.
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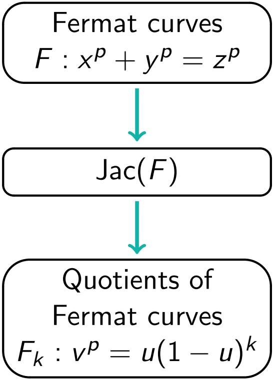
Theorem (Irokawa & Sasaki, 1995)

Assume that (X, σ) is not isomorphic to $(H_\lambda, \tau_\lambda)$ for any λ with $g = \frac{1}{2}(N-2)$ and g even. Then (X, σ) is isomorphic to $v^p = u^r(1-u)^s$, together with the automorphism $(u, v) \mapsto (u, \zeta_N v)$.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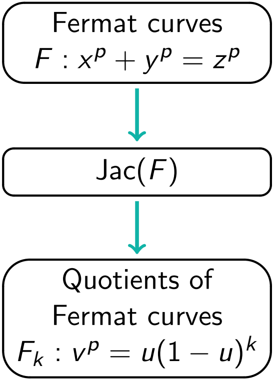

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Theorem (Gross & Rohrlich, 1978)

If $p > 7$ and $k \neq 1, (p-1)/2, p-2$, then $\mathrm{Jac}(F_k)$ has a point of infinite order.

To be continued...

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Thank you!