Introduction to Computations in Magma

Math 150 - Fall 2022

Some useful links: Documentation, general examples, our examples.
How do I get help?

- To access the documentation of a function, write `FunctionName`;.
- You can write the first letter of a function and tab complete twice to get a list of possible functions.
- You can try `ListSignatures(Type(YourElement));` to list all Magma functions that accept `YourElement` as input.

Warning! (A list from Drew Sutherland)

- Every command must end with a semicolon. Nothing happens until Magma sees the semicolon.
- Magma is fussy about types (integers and rational number are different types, for example) and elements may need to be explicitly coerced to have the type you want. To change the rational number (which will simply produce when printed) into the integer , use `Integers()!r`.
- The function `Discriminant(K)` returns the discriminant of the polynomial used to define, not the discriminant of K. Use `Discriminant(RingOfIntegers(K))` to get the discriminant of K.

Random knowledge

- To load a file, use `Load("fileAddress");`.
- To kill a process, use `control` + `c`. To kill Magma, do this twice.
- To ignore `>`, use `SetIgnorePrompt(true);`. This is very helpful when you are copying code from, let's say, worksheets.
- `$1` denotes the last printed result (you can also call `$2` and `$3`).
- The rational numbers are not a number field in Magma. You can instead call `RationalsAsNumberField();`.
- When you are debugging functions, you can `SetDebugOnError(true);`. This will give you access to the "inside" of your function, up to where Magma got stuck. You print things by `p whateverYouWant`. Use `q` to quit back to the Magma terminal. You can see all the functions that were used in your computation using `bt` (shows you the "frames"). Go to a specific frame by writing `f theNumberYouWant`. Warning: do not use `;`.

- In your home directory you can create a `file.Magmarc`, if does not exist, to have certain commands to run on start. For example, you can set `QQ := Rationals();`
- Use `control` + `e` to get to the last character of a line in the terminal. Use `control` + `a` for the first one. Do `control` + `k` to delete the line. You can also find other combinations here. Thanks Pim Spelier and Avi Kulkarni.
- Do `%p` to printing all the Magma session. Thanks Avi Kulkarni.
- You can search only for signatures without inheritance:

  `ListSignatures(ModFrmHilElt : Isa := false);`. Moreover, you can also just look for functions where your type is an argument or a return values (very useful when you try to find a function producing the type that another function needs...):

  `ListSignatures(ModFrmHilElt : Search := "ReturnValues", Isa := false);`.

  Thanks Eran Assaf!

Do you have more random knowledge? Please email it to me and I will add it to this file!

We follow Claus Fieker - Applications of the Class Field Theory of Global Fields.

Scavenger hunt

We will find a positive integer n.

1. Find k, the unique number field such that
   - $k/\mathbb{Q}$ has degree 3;
   - $k/\mathbb{Q}$ is Galois;
   - 3 is ramified in $k/\mathbb{Q}$.
   - The class group of k is trivial.

   Hint: use the LMFDB!

2. Let I be the ideal of k given by the product of the first 3 prime numbers that are inert in $k/\mathbb{Q}$. For a list of primes in Magma, try `PrimesUpTo(value);`.

3. Find the ray class group of I. Remember: `R, mR := RayClassGroup(I);`

4. Find all subgroups of R of index 2. You can use, for example:

   |L := Subgroups(R : Quot := [2]);|

5. Find all Abelian extensions K_U of k corresponding to the quotients R/U for all the subgroups U that you found in the previous step.

6. Let v be list of discriminants of all the extensions from the previous step, ordered in ascending order. You can try `Sort(List);`.

7. Compute vv, the dot product of $\langle v, v \rangle$. Maybe try `Matrix([v])*Transpose(Matrix([v]));`, but see if you can do something cleaner.

8. Compute

$$n := (10 \times \text{Number of prime divisors of } vv) + \text{Smallest prime not dividing } vv.$$